Integration by Substitution.

GAMMA
CAREER CLASSES

Important

Find all the anti Derivative of the following (1 to 4) Functions:

1. i) $(ax+b)^3$

$$I = \int (ax+b)^3 \, dx$$

$$= \int (ax+b)^3 \, dx$$

Let $ax+b=t$

$$a+0 = \frac{dt}{dx}$$

$$adx = dt$$

$$dx = \frac{dt}{a}$$

$$= \int t^3 \frac{dt}{a}$$

$$= \frac{1}{a}\int t^3 dt = \frac{1}{a}\frac{t^4}{4} + c$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$$I = \frac{1}{a}\frac{(ax+b)^4}{4} + c$$

ii) $4e^{2x} + 1$

$$\int e^x dx = e^x + c$$

$$I = \int (4e^{2x} + 1)\, dx$$

$$= 4\int e^{2x} dx + \int 1 \cdot dx$$

let $2x = t$

$$2 = \frac{dt}{dx}$$

$$dx = \frac{dt}{2}$$

$$= \frac{4}{2}\int e^t dt + x + c$$

$$= 2e^t + x + c$$

$$I = 2e^{2x} + x + c$$

2. i) $\frac{1}{\sqrt{7-2x}}$

let $I = \int \frac{1}{\sqrt{7-2x}}\,dx$

let $7-2x = t$

d.w.r. to x

$0-2 = \frac{dt}{dx}$

$-2\,dx = dt$

$dx = -\frac{dt}{2}$

$= \frac{-1}{2}\int \frac{1}{\sqrt{t}}\,dt$

$= -\frac{1}{2}\int (t)^{-\frac{1}{2}}\,dt$

$= -\frac{1}{2}\left[\frac{t^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\right]+c$ $= -\frac{1}{2}\left[\frac{t^{1/2}}{1/2}\right]+c$

$= -\frac{1}{2}\times 2(t)^{1/2}+c$ $= -(7-2x)^{1/2}+c$

3. i) $tan^2(2x-3)$

$$I = \int \tan^2(2x-3)\,dx$$

$$= \int [\sec^2(2x-3) - 1]\,dx$$

$$= \int \sec^2(2x-3)\,dx - \int 1\cdot dx$$

$$= \int \sec^2 t \,\frac{dt}{2} - x + c$$

$$= \frac{1}{2}\int \sec^2 t\,dt - x + c$$

$$= \frac{1}{2}\tan t - x + c$$

$$\boxed{\sec^2\theta = 1 + \tan^2\theta}$$

$$\sec^2\theta - 1 = \tan^2\theta$$

let $2x-3 = t$

$$2 = \frac{dt}{dx}$$

$$dx = \frac{dt}{2}$$

$$\boxed{I = \frac{1}{2}\tan(2x-3) - x + c}$$

ii) $5\text{cosec}^2(2-7x)$

$$I = \int 5\,\text{Cosec}^2(2-7x)\,dx$$

$$= 5\int \text{Cosec}^2(2-7x)\,dx$$

$$= -\frac{5}{7}\int \text{Cosec}^2 t\,dt$$

$$= +\frac{5}{7}\,\text{Cot}^2 t + C$$

$$\boxed{I = \frac{5}{7}\,\text{Cot}^2(2-7x) + C}$$

$$2-7x = t$$

$$0-7 = \frac{dt}{dx}$$

$$dx = -\frac{dt}{7}$$

6.) i) $\int \sqrt{1+\cos x}\ dx\ , 0 < x < \pi$

$$I = \int \sqrt{1+\cos x}\ dx$$

$$= \int \sqrt{1+2\cos^2\frac{x}{2}-1}\ dx$$

$$= \int \sqrt{2\cos^2\frac{x}{2}}\ dx$$

$$= \sqrt{2}\int \cos\frac{x}{2}\ dx$$

$$= 2\sqrt{2}\int \cos t\ dt = 2\sqrt{2}(-\sin t)+c$$

$$-2\sqrt{2}\sin\frac{x}{2}+c$$

$\cos x = 2\cos^2\frac{x}{2}-1$

$\cos x = 1-2\sin^2\frac{x}{2}$

Let

$\frac{x}{2} = t$

$\frac{1}{2} = \frac{dt}{dx}$

$\frac{1}{2}dx = dt$

$dx = 2dt$

ii) $\int sec^2 x\, cosec^2 x\, dx$

$$I = \int Sec^2 x\, Cosec^2 x\, dx$$

$$= \int \frac{1}{Cos^2 x\, Sin^2 x}\, dx$$

$$= \int \frac{Sin^2 x + Cos^2 x}{Cos^2 x\, Sin^2 x}\, dx$$

$$= \int \frac{\cancel{Sin^2 x}}{Cos^2 x \cancel{Sin^2 x}}\, dx + \int \frac{\cancel{Cos^2 x}}{\cancel{Cos^2 x}\, Sin^2 x}\, dx$$

$$\int \frac{1}{Cos^2 x}\, dx + \int \frac{1}{Sin^2 x}\, dx$$

$$= \int Sec^2 x\, dx + \int Cosec^2 x\, dx$$

$$I = \tan x - \cot x + C$$

$$Sin^2 x + Cos^2 x = 1$$

ii) $\int \frac{x+2}{3x^2+5x-2}\,dx$

$$I=\int \frac{x+2}{3x^2+5x-2}\,dx$$

$$=\int \frac{x+2}{3x^2+(6-1)x-2}\,dx$$

$$=\int \frac{x+2}{3x^2+6x-x-2}\,dx$$

$$=\int \frac{x+2}{3x(x+2)-1(x+2)}\,dx$$

$$\int \frac{x+2}{(x+2)(3x-1)}\,dx$$

$$=\int \frac{1}{3x-1}\,dx$$

$=\frac{1}{3}\int \frac{1}{t}\,dt$ put $3x-1=t$

$3dx=dt$

$dx=\frac{dt}{3}$

$=\frac{1}{3}\log|t|+c$

$$I=\frac{1}{3}\log|3x-1|+c$$

10.i) $\int \frac{x}{(x+1)^2}\,dx$

$$I = \int \frac{x}{(x+1)^2}\,dx$$

$$= \int \frac{x+1-1}{(x+1)^2}\,dx$$

$$= \int \frac{x+1}{(x+1)^2}\,dx - \int \frac{1}{(x+1)^2}\,dx$$

$$= \int \frac{1}{(x+1)}\,dx - \int \frac{1}{(x+1)^2}\,dx$$

$$\log(x+1) - \int (x+1)^{-2}\,dx + c$$

$$\log(x+1) - \frac{(x+1)^{-1}}{-1} + c$$

$$I = \log(x+1) + \frac{1}{(x+1)} + c$$

ii) $\int \frac{x^2+2}{x+1}\,dx$

$$I = \int \frac{x^2+2}{x+1}\,dx$$

$$= \int \frac{x^2-1+3}{x+1}\,dx$$

$$= \int \frac{(x^2-1)+3}{(x+1)}\,dx$$

$$= \int \frac{(x^2-1)}{x+1}\,dx + \int \frac{3}{(x+1)}\,dx$$

$$= \int \frac{(x-1)(x+1)}{(x+1)}\,dx + \int \frac{3}{(x+1)}\,dx$$

$$= \int (x-1)\,dx + \int \frac{3}{(x+1)}\,dx$$

$$= \frac{x^2}{2} - x + 3\log|x+1| + C$$